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USE OF LOGARITHMS AND LOGARITHMIC TABLES

SECOND EDITION

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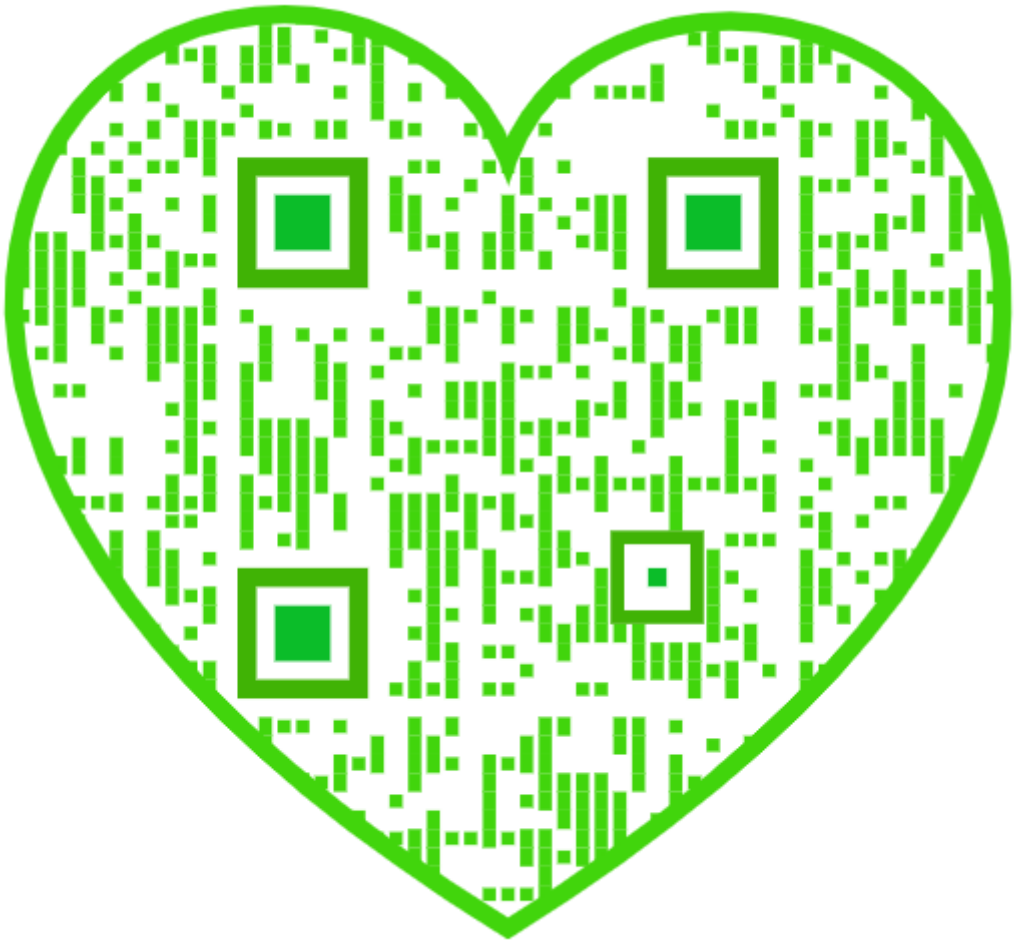

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THE USE OF LOGARITHMS

It is not intended in the following pages to discuss the mathematical principles on which logarithms and expressions containing logarithms are based, but simply to impart a working knowledge of the use of logarithms, so that practical men, unfamiliar with this means for eliminating much of the work ordinarily required in long and cumbersome calculations, may be able to make advantageous use of the tables of logarithms given in the latter part of the book.

The object of logarithms is to facilitate and shorten calculations involving multiplication, division, the extraction of roots, and the obtaining of powers of numbers, as will be explained later; but ordinary logarithms cannot be applied to operations involving addition and subtraction. Before entering directly upon the subject of the use of logarithms in carrying out the various classes of calculations mentioned, it will be necessary to deal with the question of how to find the logarithm of a given number from the tables; or, if the logarithm is given, how to find the corresponding number.

A logarithm consists of two parts, a whole number and a decimal. The whole number, which may be either a positive or a negative number,* or zero, according to the rules which will be given in the following, is called the *characteristic;* the decimal is called the *mantissa.* The decimal or mantissa only is given in the tables of logarithms on pages 18 to 35, inclusive, and is always positive. The logarithm of 350, for example is 2.54407. Here "2" is the characteristic, and "54407" is the mantissa, this latter being found from the table on page 23.

Rules for Finding the Characteristic

The characteristic is not given in the tables of logarithms, due partly to the fact that it can so easily be determined without the aid of tables, and partly because the tables, so to say, are universal when the characteristic is left out.

For 1 *and all numbers greater than* 1 *the characteristic is one less than the number of places to the left of the decimal point in the given number.*

The characteristic of the logarithm of 237, therefore, is 2, because 2 is one less than the number of figures in 237. The characteristic of the logarithm of 237.26 is also 2, because it is only the number of figures to the left of the decimal point that is considered.

The characteristic of the logarithm of 7 is 0, because 0 is one less

* See MACHINERY'S Reference Series No. 54, Solution of Triangles, Chapter III, Positive and Negative Quantities.

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than 1, which is the number of places in the given number. Below are given several numbers with the characteristics of their logarithms:

Number	Characteristic of Logarithm
31	1
3163	3
229.634	2
1,112,352.62	6
1000	3
100	2
1	0

For numbers smaller than 1, *that is, for numbers wholly decimal, the characteristic is negative, and its numerical value is one more than the number of ciphers between the decimal point and the first decimal which is not a cipher.*

The numerical value of the characteristic of the logarithm of 0.036, therefore, is 2; being negative it is — 2. Instead of writing the minus sign (—) in front of or before the figure (— 2), it is, however, written over the figure ($\bar{2}$). This method is used because the minus sign refers only to the characteristic, and not to the mantissa, this latter always being positive. In the same way, the characteristic of the logarithm of 0.36 is $\bar{1}$; and the characteristic of the logarithm of 0.0006 is $\bar{4}$. Below are given several examples:

Number	Characteristic of Logarithm
0.0000375	$\bar{5}$
0.3	$\bar{1}$
0.375	$\bar{1}$
0.000812	$\bar{4}$
0.01234	$\bar{2}$

Finding the Logarithms of Numbers

After the characteristic has been found by the rules just given, the mantissa must be found from the tables of logarithms. When finding the mantissa the decimal point is entirely disregarded. The mantissa of the logarithms of 2716, 271.6, 27.16, 2.716, or 0.02716, for example, is the same; it is only necessary to find the given figures in the tables, irrespective of the location of the decimal point.

Referring now to the tables on pages 18 to 35, it will be seen that numbers from 100 to 1,000 are given in the left-hand column. In addition, at the top of the tables, are figures from 0 to 9, each heading a column of logarithms. These additional figures make it possible to obtain directly from the tables the logarithms for all numbers with four figures or less. The body of the tables gives the mantissa of the logarithms.

While the tables do not give directly the mantissa of logarithms of numbers with more than four figures, it is possible to approximate the logarithm for numbers with a greater number of figures by methods which will be explained later. At present, when the logarithm is required for numbers with five or more figures, we will assume that

for practical results it is accurate enough to find the mantissa of the logarithm of the first four figures of the number, remembering, of course, that if the fifth figure is more than 5, then the fourth figure should be increased by one unit.

To find the logarithm of a number from the tables, first locate the three first figures of the number for which the logarithm is required in the left-hand column, and then find the fourth figure at the top of the columns of the page. Then follow the column down from this last figure until opposite the three first figures in the left-hand column. The figure thus found in the body of the table is the mantissa of the logarithm, the characteristic having already been found by the rules previously given.

If the number of which the logarithm is required does not contain four figures, annex ciphers to the right so as to obtain four figures. If the mantissa of the logarithm of 6 is required, for example, find the mantissa for 6,000. The mantissa is the same for 6, 0.6, 0.06, 60, 600, 6,000, etc., as already explained. The difference in the logarithm is taken care of by the change of characteristic for these various values. In order to save space in the tables, it will be seen by referring to them that the first two figures of the mantissa have been given in the first columns of logarithms only, the 0-column. These two figures should, however, always precede the three figures given in each of the following columns.

A few examples will now make the use of the tables clearer.

Example 1.—Find the logarithm of 1852.

Following the rule already given, locate 185 in the left-hand column of the tables (it will be found on page 19), and then following downward the column headed "2" at the top of the page, find the required mantissa opposite 185. It will be seen that the mantissa is .26764,* the figures 26 being found in the column under "0" and prefixed to the figures 764 found directly in the column under "2." The characteristic of the logarithm, according to the rules previously given, is 3. Hence the logarithm of 1852, or, as it is commonly written, log 1852 = 3.26764.

Example 2.—Find log 1.852.

As the figures in this number are the same as in that given in example 1, the mantissa remains the same; but the characteristic is 0. Therefore, the required logarithm, or log 1.852 = 0.26764.

Example 3.—Find log 93.14.

Locate 931 in the left-hand column of the tables (page 34), and then following downward the column headed "4" at the top of the page, find the required mantissa opposite 931. It will be found that the mantissa is .96914. The characteristic is 1. Hence log 93.14 = 1.96914.

Example 4.—Find log 4.576.

Find as before the last three figures of the mantissa opposite 457

* All the mantissas, or the numbers in the tables, are decimals, and the decimal point has, therefore, been omitted entirely, since no confusion could arise from this; but it should always be put before the figures of the mantissa as soon as taken from the table. The practice of eliminating the decimal point from the tables is common to all logarithmic tables.

in the left-hand column, and in the column under "6" at the top of the page. The figures are *049. The sign * indicates that the two figures to be prefixed are not 65, as would ordinarily be the case, but 66, or the figures given in the next following line in the 0-column. This rule should always be borne in mind. Hence, log 4.576 = 0.66049.

Example 5.—Find log 72.

To find the mantissa, proceed as if it were to be found for 7200. This we find from the tables to be .85733. The characteristic of the logarithm of 72 is 1. Hence log 72 = 1.85733.

Example 6.—Find log 0.007631.

To find the mantissa, proceed as if it were to be found for 7631. This we find from the tables to be .88258. The characteristic is $\bar{3}$, according to the rule given for characteristics of logarithms of numbers less than 1. Hence, log 0.007631 = $\bar{3}$.88258.

Example 7.—Find log 37,262.

While we will later explain how to find more exactly the mantissa for a number with five figures, at present we may consider it accurate enough for our purpose to find the mantissa for four figures, or for 3726. This is .57124. The characteristic of the logarithm of 37,262 is 4. Hence log 37,262 = 4.57124. This, of course, is only an approximation, but is near enough for nearly all shop and general engineering calculations.

If the given number had been 37,267 instead of 37,262, the logarithm should have been found for 3727, as the fourth figure then should have been increased by 1, when dropping the fifth figure, which is larger than $\bar{5}$.

Below are given several examples of numbers with their logarithms. A careful study of these examples, the student finding the logarithms for himself from the tables, and checking them with the results given, will tend to make the methods employed clearer and fix them in the mind.

Number	Logarithm
16.95	1.22917
2	0.30103
966.2	2.98507
151	2.17898
3.5671	0.55230
12.91	1.11093
3803.8	3.58024
0.007	$\bar{3}$.84510

It should be understood that in logarithms of numbers less than 1, the characteristic, only, is negative. The mantissa is always positive, so that $\bar{3}$.84510 actually means (— 3) + 0.84510.

Finding the Number whose Logarithm is Given

When a logarithm is given, and it is required to find the corresponding number, first find the first two figures of the mantissa in the column headed "0" in the tables. Then find in the group of mantissas, all having the same first two figures, the remaining three figures.

These may be in any of the columns headed "0" to "9." The number heading the column in which the last three figures of the mantissa were found, is the last figure in the number sought, and the number in the left-hand column, headed "N," in line with the figures of the mantissa, gives the three first figures in the number sought.

When the actual figures in the number sought have thus been determined, locate the decimal point according to the rules given for the characteristic of logarithms. If the characteristic is greater than 3, ciphers are added. For example, if the figures corresponding to a certain mantissa are 3765, and the characteristic is 5, then the number sought must have 6 figures to the left of the decimal point, and hence would be 376500. If the characteristic had been $\bar{3}$, then the number sought, in this case, would have been 0.003765.

If the mantissa is not exactly obtainable in the tables, find the nearest mantissa in the table to the one given, and determine the number corresponding to this. In most cases this gives ample accuracy. A method will be explained later whereby still greater accuracy may be obtained, but for the present it will be assumed that the numbers corresponding to the nearest mantissa in the tables are accurate enough for practical purposes.

A few examples will now be given in which it is required to find the number when the logarithm is given.

Example 1.—Find the number whose logarithm is 3.89382.

First find the first two figures of the mantissa (89) in the column headed "0" in the tables. Then find the remaining three figures (382) in the mantissas which all have 89 for their first two figures. The figures "382" are found in the column headed "1," which thus is the last figure in the number sought; the figures "382" are also opposite the number 783 in the left-hand column, which gives the first three figures in the number sought. The figures in this number, thus, are 7831, and as the characteristic is 3, it indicates that there are four figures to the left of the decimal point, or, in other words, that 7831 is a whole number.

Example 2.—Find the number whose logarithm is 2.75020.

First find the first two figures of the mantissa (75) in the column head "0" in the tables. Then find the remaining three figures (020) in the mantissas, which all have 75 for their first two figures. The * in front of the figure *020 in the line next above that in which 75 was found indicates that these figures belong to the group preceded by 75. Therefore, as *020 is found in the column headed "6" and opposite the number 562 in the left-hand column, the figures in the number required to be found are 5626. As the characteristic is 2, the decimal point is placed after the first three figures, and, hence, the number whose logarithm is 2.75020 is 562.6.

Example 3.—Find the number whose logarithm is 2.45350.

After having located 45 in the column headed "0," it will be found that the last three figures (350) of the mantissa are not to be found in the table in the group preceded by 45. The nearest value in the table, which is 347, is, therefore, located, and the corresponding num-

ber is found to be 284.1, the decimal point being placed after the third figure, because the characteristic of the logarithm is 2. Had the characteristic of the logarithm been 5 instead of 2, the number to be found would have been 284,100.

Below are given a selection of examples of logarithms with their corresponding numbers. The student should find the numbers for himself from the tables, and check them with the results given. This will aid in fixing the rules and methods employed more firmly in the mind.

Logarithm	Corresponding Number
1.43201	27.04
4.89170	77,930
$\bar{2}$.76057	0.05762
0.12096	1.321
2.99099	979.5
$\bar{1}$.60206	0.4
5.60206	400,000

It being now assumed that the student has mastered the methods for finding the logarithms for given numbers, and the numbers for given logarithms, from the tables, the use of logarithms in multiplication and division will next be explained.

Multiplication by Logarithms

If two or more numbers are to be multiplied together, find the logarithms of the numbers to be multiplied, and then add these logarithms; the sum is the logarithm of the product, and the number corresponding to this logarithm is the required product.

Example 1.—Find the product of $2831 \times 2.692 \times 29.69 \times 19.4$.

This calculation is carried out by means of logarithms as follows:

$$\begin{aligned} \log\ 2831. &= 3.45194 \\ \log\ 2.692 &= 0.43008 \\ \log\ 29.69 &= 1.47261 \\ \log\ 19.4 &= 1.28780 \\ \hline & 6.64243 \end{aligned}$$

The sum of the logarithms, 6.64243, is the logarithm of the product, and from the tables we then find that the product equals 4,390,000. This result is, of course, only approximately correct, at the last three figures are added ciphers; but for most engineering calculations the result would give all the accuracy required. In most engineering calculations one or more factors are *assumed* from experimental values, and as these assumed values evidently must often vary between wide limits, it would show lack of judgment to require calculations in which such assumed values enter, to be carried out with too many "significant" figures. Such values are fully as well expressed in round numbers, with ciphers annexed to give the required value to the figures found from the tables.

If one or more of the characteristics of the logarithms are negative, these are subtracted instead of added to the sum of the character-

istics. The mantissas, as already mentioned, are always positive, so that they are always added in the usual manner. In order to fully understand the adding of positive and negative numbers in the following examples, the student should be familiar with calculations with positive and negative quantities, as explained in MACHINERY'S Reference Series No. 54, Solution of Triangles, Chapter III.

Example 2.—Find the product $371.2 \times 0.0972 \times 3$.

$$\begin{aligned} \log\ 371.2 &= 2.56961 \\ \log\ 0.0972 &= \bar{2}.98767 \\ \log\ 3. &= 0.47712 \\ \hline & 2.03440 \end{aligned}$$

The number corresponding to the logarithm 2.03440 is 108.2. Note that the first two figures of the mantissa of the logarithm are 03.

Example 3.—Find the product $12.76 \times 0.012 \times 0.6$.

$$\begin{aligned} \log\ 12.76 &= 1.10585 \\ \log\ 0.012 &= \bar{2}.07918 \\ \log\ 0.6 &= \bar{1}.77815 \\ \hline & \bar{2}.96318 \end{aligned}$$

The product, hence, is 0.09187.

Division by Logarithms

When dividing one number by another, the logarithm of the divisor is subtracted from the logarithm of the dividend. The remainder is the logarithm of the quotient.

For example, if we are to find the quotient of $7568 \div 935.3$, we first find log 7568 and then subtract from it log 935.3. The remainder is then the logarithm of the quotient.

It is advisable, however, to make a modification, as explained in the following, of the logarithm of the divisor so as to permit of its addition to, instead of its subtraction from the logarithm of the dividend. Assume, for instance, that an example, as below, were given:

$$\frac{375.2 \times 97.2 \times 0.0762 \times 3}{962.1 \times 92 \times 33.26}$$

It would be perfectly correct to find the logarithms of all the factors in the numerator and add them together, and then the logarithms of all the factors in the denominator and add them together; and finally subtract the sum of the logarithms of the denominator from the sum of the logarithms of the numerator. The remainder is the logarithm of the result of the calculation. This method, however, involves two separate additions and one subtraction. It is possible, by a modification of the logarithms of the numbers in the denominator to so arrange the calculation that a single addition will give the logarithm of the final result.

In dealing with positive and negative numbers we learn that if we add a negative number to a positive number, the sum will be the same as if we subtract the numerical value of the negative

number from the positive number; that is $5 + (-2) = 5 - 2 = 3$. If we reverse this proposition we have $5 - 2 = 5 + (-2)$. If we now assume that 5 is the logarithm of a certain number a and 2 the logarithm of another number b, and if we insert these values in the last expression, instead of 5 and 2, we have:

$$\log a - \log b = \log a + (-\log b).$$

From this we see that instead of subtracting log b from log a we can add the negative value of log b and obtain the same result. As the mantissa always must remain positive, in order to permit direct addition, the negative value of the logarithm cannot be obtained by simply placing a minus sign before it. Instead, it is obtained in the following manner:

If the characteristic is positive, add 1 to its numerical value and place a minus sign over it. To obtain the mantissa, subtract the given mantissa from 1.00000.

Example 1.—The logarithm of 950 = 2.97772. Find ($-\log 950$).

According to the rule given, the characteristic will be $\bar{3}$. The mantissa will be 1.00000 — .97772 = .02228. The last calculation can be carried out mentally without writing it down at all, by simply finding the figure which, added to the last figure in the given mantissa would make the sum 10, and the figures which added to each of the other figures in the mantissa, would make the sum 9. as shown below:

9	7	7	7	2
0	2	2	2	8
9	9	9	9	10

As this calculation is easily carried out mentally, the method described, when fully mastered, greatly simplifies the work where operations of both multiplication and division are to be performed in the same example.

Example 2.—The logarithm of 2 is 0.30103. Find ($-\log 2$).

According to the given rules the characteristic is $\bar{1}$, and the mantissa, .69897.

The following examples should be studied until thoroughly understood:

log 270.	= 2.43136	— log 270.	= $\bar{3}$.56864
log 10.	= 1.00000	— log 10.	= $\bar{1}$.00000
log 26.99	= 1.43120	— log 26.99	= $\bar{2}$.56880

In the example in the second line an exception from the rule for obtaining the mantissa of the negative logarithm is made. It is obvious, however, that if log 10 = 1.00000, then ($-\log 10$) = $\bar{1}$.00000. In the example in the last line there is another deviation from the literal understanding of the rule for the mantissa. As the last figure in the positive logarithm is 0, the last figure in ($-\log$ 26.99) is also 0, and the next last figure is treated as if it were the last, making the next last figure in the negative logarithm 8.

If the characteristic of the logarithm is negative, subtract 1 from its numerical value, and make it positive. The mantissa is obtained by the same rule as before.

Example 1.—The logarithm of 0.003 = $\bar{3}$.47712. Find (— log 0.003). According to the rule just given the characteristic will be 2. The mantissa will be .52288. Hence (— log 0.003) = 2.52288.

The following examples should be studied until fully understood:

log 0.3	= $\bar{1}$.47712	— log 0.3	= 0.52288
log 0.0006963	= $\bar{4}$.84280	— log 0.0006963	= 3.15720
log 0.6607	= $\bar{1}$.82000	— log 0.6607	= 0.18000

When sufficient practice has been obtained, the negative value of a logarithm can be read off almost as quickly from the tables as the positive value given, and the subsequent gain of time, and the ease of the calculations following, more than justify this short-cut method.

Examples of the Use of Logarithms

We will now give a number of examples of the use of logarithms in calculations involving multiplication and division. No comments will be made, as it is assumed that the student has now grasped the principles sufficiently to be able to follow the methods used without further explanation.

Example 1.

$$\frac{0.0272 \times 27.1 \times 12.6}{2.371 \times 0.007}$$

log	0.0272	= $\bar{2}$.43457
log	27.1	= 1.43297
log	12.6	= 1.10037
— log	2.371	= $\bar{1}$.62507
— log	0.007	= 2.15490
		2.74788

The result, then, is 559.6.

Example 2.

$$\frac{0.3752 \times 0.063 \times 0.012}{0.092 \times 1289}$$

log	0.3752	= $\bar{1}$.57426
log	0.063	= $\bar{2}$.79934
log	0.012	= $\bar{2}$.07918
— log	0.092	= 1.03621
— log	1289.0	= $\bar{4}$.88975
		$\bar{6}$.37874

The result, then, is 0.000002392.

Example 3.

$$3.463 \times 1.056 \times 14.7 \times 144 \times 10$$

log	3.463	= 0.53945
log	1.056	= 0.02366
log	14.7	= 1.16732
log	144.0	= 2.15836
log	10.0	= 1.00000
		4.88879

The result, then, is 77,410.

Example 4.

$$0.00005427 \times 392 \times 2.5 \times 200 \times 200$$

$$\begin{aligned}
\log\ 0.00005427 &= \bar{5}.73456 \\
\log\ 392. &= 2.59329 \\
\log\ 2.5 &= 0.39794 \\
\log\ 200. &= 2.30103 \\
\log\ 200. &= 2.30103 \\
&\overline{\qquad\qquad} \\
&3.32785
\end{aligned}$$

Hence, the result is 2127.

Obtaining the Powers of Numbers

Expressions of the form 6.51^3 can easily be calculated by means of logarithms. The small (^{3}) is called exponent.* In this case the "third power" of 6.51 is required.

A number may be raised to any power by simply multiplying the logarithm of the number by the exponent of the number. The product gives the logarithm of the value of the power.

Example 1.—Find the value of 6.51^3.

$$\begin{aligned}
\log\ 6.51 &= 0.81358 \\
3 \times 0.81358 &= 2.44074
\end{aligned}$$

The logarithm 2.44074 is then the logarithm of 6.51^3. Hence 6.51^3 equals the number corresponding to this logarithm, as found from the tables, or $6.51^3 = 275.9$.

Example 2.—Find the value of $12^{1.29}$.

$$\begin{aligned}
\log\ 12 &= 1.07918 \\
1.29 \times 1.07918 &= 1.39214
\end{aligned}$$

Hence, $12^{1.29} = 24.67$.

The multiplication 1.29×1.07918 is carried out in the usual arithmetical way. The example above is one of a type which cannot be solved by any means except by the use of logarithms. An expression of the form 6.51^3 can be found by arithmetic by multiplying $6.51 \times 6.51 \times 6.51$, but an expression of the form $12^{1.29}$ does not permit of being calculated by any arithmetical method. Logarithms are here absolutely essential.

One difficulty is met with when raising a number less than 1 to a given power. The logarithm is then composed of a negative term, the characteristic, and a positive term, the mantissa. For example: Find the value 0.31^5. The logarithm of $0.31 = \bar{1}.49136$. In this case, multiply, separately, the characteristic and the mantissa by the exponent, as shown below. Then add the products.

$$\log\ 0.31 = \bar{1}.49136$$

Multiplying characteristic and mantissa separately by 5 we have:

$$\begin{aligned}
5 \times \bar{1} &= \bar{5} \\
5 \times .49136 &= 2.45680 \\
&\overline{\qquad\qquad} \\
\log\ 0.31^5 &= \bar{3}.45680
\end{aligned}$$

Hence, $0.31^5 = 0.002863$.

* See MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, Chapter III.

If the exponent is not a whole number, the procedure will be somewhat more complicated. The principle of the method, however, remains the same.

Example: Find the value of $0.06^{2.31}$

$$\log 0.06 = \bar{2}.77815$$

Then

$$2.31 \times \bar{2} = 2.31 \times (-2) = -4.62$$
$$2.31 \times 0.77815 = 1.79753$$

In this case, the first product, — 4.62, is negative both as regards the whole number and the decimal. In order to make the decimal positive so that we may be able to add it directly to the second product, 1.79753, we must use the same rule as given for changing a logarithm with a positive characteristic to a negative value. Hence $-4.62 = \bar{5}.38$. We can now add the products:

$$\bar{5}.38$$
$$1.79753$$
$$\log 0.06^{2.31} = \bar{3}.17753$$

Hence $0.06^{2.31} = 0.001505$.

As a further example, find $0.07^{3.51}$.

$$\log 0.07 = \bar{2}.84510$$

Then

$$3.51 \times \bar{2} = 3.51 \times (-2) = -7.02 = \bar{8}.98$$
$$3.51 \times .84510 = 2.96630$$
$$\log 0.07^{3.51} = \bar{5}.94630$$

Hence $0.07^{3.51} = 0.00008837$.

Extracting Roots by Logarithms

Roots of numbers, as for example $\sqrt[5]{37}$, can easily be extracted by means of logarithms. The small (5) in the radical ($\sqrt{}$) of the root-sign is called the index of the root. In the case of the square root the index is (2), but it is not usually indicated, the square root being merely expressed by the sign $\sqrt{}$.

Any root of a number may be found by dividing its logarithm by the index of the root; the quotient is the logarithm of the root.

Example 1.—Find $\sqrt[3]{276}$.

$$\log 276 = 2.44091$$
$$2.44091 \div 3 = 0.81364$$

Hence $\log \sqrt[3]{276} = 0.81364$, and $\sqrt[3]{276} = 6.511$.

Example 2.—Find $\sqrt[3]{0.67}$.

$$\log 0.67 = \bar{1}.82607$$

In this case we cannot divide directly, because we have a negative characteristic and a positive mantissa. We then proceed as follows: Add numerically as many negative units or parts of units to the characteristic as is necessary to make it evenly contain the index of the root. Then add the *same* number of *positive* units or parts of units to the mantissa. Divide each separately by the index. The quotients give

the characteristic and mantissa, respectively, of the logarithm of the root.

Proceeding with the example above according to this rule, we have:

$$\bar{1} + \bar{2} = \bar{3}; \quad \bar{3} + 8 = \bar{1}.$$

$$.82607 + 2 = 2.82607; \quad 2.82607 \div 3 = .94202.$$

Hence, $\log \sqrt[3]{0.67} = \bar{1}.94202$, and $\sqrt[3]{0.67} = 0.875$.

Example 3.—Find $\sqrt[1.7]{0.2}$.

$$\log 0.2 = \bar{1}.30103.$$

If we add (-0.7) to the characteristic of the logarithm found, it will be evenly divisible by the index of the root.

Hence:

$$\bar{1} + (-0.7) = -1.7; \quad -1.7 \div 1.7 = \bar{1}.$$

$$.30103 + 0.7 = 1.00103; \quad 1.00103 \div 1.7 = .58884.$$

Hence, $\log \sqrt[1.7]{0.2} = \bar{1}.58884$, and $\sqrt[1.7]{0.2} = 0.388$.

A number of examples of the use of logarithms in the solution of everyday problems in mechanics, are given in MACHINERY'S Reference Series No. 19, Use of Formulas in Mechanics, Chapter II, 2nd edition.

When exponents or indices are given in common fractions, it is usually best to change them to decimal fractions before proceeding further with the problem.

Interpolation

If the number for which the logarithm is required consists of five figures, it is possible, by means of the small tables in the right-hand column of the logarithm tables, headed "P. P." (proportional parts), to obtain the logarithm more accurately than by taking the nearest value for four figures, as has previously been done in the examples given. The method by which the logarithm is then obtained is called interpolation.

In the same way, if a logarithm is given, the exact value of which cannot be found in the tables, the number corresponding to the logarithm can be found to five figures by interpolation, although the main tables contain only numbers of four figures.

The logarithm of 2853 is 3.45530, and the logarithm of 2854 is 3.45545, as found from the tables. Assume that the logarithm of 2853.6 were required. It is evident that the logarithm of this latter number must have a value between the logarithms of 2853 and 2854. It must be somewhat greater than the logarithm of the former number, and somewhat smaller than that of the latter. While the logarithms, in general, are not proportional to the numbers to which they correspond, the difference is very slight in cases where the increase in the numbers is small; so that, in the case of an increase from 2853 to 2854, the logarithms for the decimals 2853.1, 2853.2, etc., may be considered proportional to the numbers. It is on this basis that the small tables in the right-hand column headed "P.P." are calculated, and the logarithm of 2853.6, for example, is found as follows:

Find first the difference between the nearest larger and the nearest smaller logarithms. Log 2854 = 3.45545 and log 2853 = 3.45530. The

difference is 0.00015. Then in the small table headed "15" in the right-hand column find the figure opposite 6 (6 being the last or fifth figure in the given number). This figure is 9.0. Add this to the mantissa of the smaller of the two logarithms already found, disregarding the decimal point in the mantissa, and considering it, for the while being as a whole number. Then 45530 + 9.0 = 45539. This is the mantissa of the logarithm of 2853.6, and the complete logarithm is 3.45539.

Example.—Find log 236.24.

Log 236.2 = 2.37328; log 236.3 = 2.37346; difference = 0.00018. In table "18" the proportional part opposite 4 is 7.2. Then 37328 + 7.2 = 37335.2. The decimal 2 is not used, but is dropped. Hence log 236.24 = 2.37335.

If the proportional part to be added has a decimal larger than 5, it should not be dropped before the figure preceding it has been raised one unit. For example, if the logarithm of 236.26 had been required, then the proportional part would have been 10.8 and the mantissa sought 37328 + 10.8 = 37338.8. Now the decimal 8 cannot be dropped before the figure 8 preceding it has been raised to 9. Then log 236.26 = 2.37339.

If the number for which the lorgarithm is to be found consists of more than five figures, find the mantissa for the nearest number of five figures, but choose the characteristic according to the total number of figures to the left of the decimal point. For example, if the logarithm of 626,923 is required, find the mantissa, by interpolation, for 62692. If the logarithm for 626,928 is required, find the mantissa for 62693, always remembering to raise the value of the last figure, if the figure dropped is more than 5. The characteristic in each of these examples would, of course, be 5, as it is chosen according to the total number of figures to the left of the decimal point in the given numbers, which is 6.

To find a number whose logarithm is given more accurately than to four figures, when the given mantissa cannot be found exactly in the tables, find the mantissa which is nearest to, but less than the given mantissa. Subtract this mantissa from the nearest larger mantissa in the tables and find in the right-hand column the small table headed by this difference. Then subtract the nearest smaller mantissa from the given logarithm, and find the difference, exact or approximate, in the "proportional part" table (in the right-hand column of this table). The corresponding figure in the left-hand column of the "proportional part" table is the fifth figure in the number sought, the other four figures being those corresponding to the logarithm next maller to the given logarithm.

Example.—Find the number whose logarithm is 4.46262.

The mantissa can not be found exactly in the tables; therefore, following the rules just given, we see that the nearest smaller mantissa in the tables equals 46255. The next larger is 46270. The difference between them is 15. The difference between the mantissa of the given logarithm, 46262 and the next smaller mantissa, 46255 is 7. Now, in the proportional parts table opposite 7.5 in the right-hand column of

the table headed 15, we find that the fifth figure of the number sought would be 5. The four first figures are 2901. Hence the number sought is 29,015.

The following examples, if carefully studied, will give the student a clear conception of the method of interpolation.

Number	Logarithm
52,163	4.71736
26.913	1.42996
0.012635	$\bar{2}$.10157
12.375	1.09254
6.9592	0.84256

The student should find for himself, first the logarithms corresponding to the given numbers, and then the numbers corresponding to the given logarithms. In this way a check on the accuracy of the work can be obtained by comparing with the results given.

General Remarks

In the system of logarithms tabulated on pages 18 to 35, the base of the logarithms is 10; that is, the logarithm is actually the exponent which would be affixed to 10 in order to give the number corresponding to the logarithm. For example log 20 = 1.30103, which is the same as to say that $10^{1.30103} = 20$. Log 100 = 2, and, of course, we know that $10^2 = 100$. As $10^1 = 10$, the logarithm of 10 = 1. The logarithm of 1 = 0. The system of logarithms having 10 for its base is called the Briggs or the common system of logarithms.

While the accompanying logarithm tables are given to five decimals, it should be understood that the logarithm of a number can be calculated with any degree of accuracy, so that large logarithm tables give the logarithm with as many as seven decimal places, and some, used for very accurate scientific investigations, give as many as ten decimals. It will be noticed that in the accompanying tables the figure 5, when in the fifth decimal place, is either written $\dot{5}$ or $\bar{5}$. If the sixth place is 5 or more, the next larger number is used in the fifth place, and the logarithm is then written in the form 3.90855. The dash over the 5 shows that the logarithm is less than given. If the sixth figure is less than 5, the logarithm is written 3.91025, the dot over the 5 showing that the logarithm is more than given. In calculations of the type previously explained, this, however, need not be taken into consideration and these signs should be disregarded by the student.

Hyperbolic Logarithms

In certain mechanical calculations, notably those involving the calculation of the mean effective pressure of steam in engine cylinders, use is made of logarithms having for their base the number 2.7183, commonly designated *e*, and found by abstract mathematical analysis. These logarithms are termed *hyperbolic*, *Napierian* or *natural;* the preferable name, and that most commonly in use in the United States is hyperbolic logarithms. The hyperbolic logarithms are usually designated "hyp. log." Thus, when log 12 is required, it always refers to

common logarithms, but when the hyp. log 12 is required, reference is made to hyperbolic logarithms. Sometimes, the hyperbolic logarithm is also designated "$\log_e$" and "nat. log."

To convert the common logarithms to hyperbolic logarithms, the former should be multiplied by 2.30258. To convert hyperbolic logarithms to common logarithms, multiply by 0.43429. These multipliers will be found of value in cases where hyperbolic logarithms are required in formulas. Hyperbolic logarithms find extensive use in higher mathematics.

SECTION II

TABLES OF COMMON LOGARITHMS

1 TO 10,000

N.	L. 0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.			
	44	**43**	**42**
1	4,4	4,3	4,2
2	8,8	8,6	8,4
3	13,2	12,9	12,6
4	17,6	17,2	16,8
5	22,0	21,5	21,0
6	26,4	25,8	25,2
7	30,8	30,1	29,4
8	35,2	34,4	33,6
9	39,6	38,7	37,8
	41	**40**	**39**
1	4,1	4,0	3,9
2	8,2	8,0	7,8
3	12,3	12,0	11,7
4	16,4	16,0	15,6
5	20,5	20,0	19,5
6	24,6	24,0	23,4
7	28,7	28,0	27,3
8	32,8	32,0	31,2
9	36,9	36,0	35,1
	38	**37**	**36**
1	3,8	3,7	3,6
2	7,6	7,4	7,2
3	11,4	11,1	10,8
4	15,2	14,8	14,4
5	19,0	18,5	18,0
6	22,8	22,2	21,6
7	26,6	25,9	25,2
8	30,4	29,6	28,8
9	34,2	33,3	32,4
	35	**34**	**33**
1	3,5	3,4	3,3
2	7,0	6,8	6,6
3	10,5	10,2	9,9
4	14,0	13,6	13,2
5	17,5	17,0	16,5
6	21,0	20,4	19,8
7	24,5	23,8	23,1
8	28,0	27,2	26,4
9	31,5	30,6	29,7
	32	**31**	**30**
1	3,2	3,1	3,0
2	6,4	6,2	6,0
3	9,6	9,3	9,0
4	12,8	12,4	12,0
5	16,0	15,5	15,0
6	19,2	18,6	18,0
7	22,4	21,7	21,0
8	25,6	24,8	24,0
9	28,8	27,9	27,0

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	29	28
1	2,9	2,8
2	5,8	5,6
3	8,7	8,4
4	11,6	11,2
5	14,5	14,0
6	17,4	16,8
7	20,3	19,6
8	23,2	22,4
9	26,1	25,2

	27	26
1	2,7	2,6
2	5,4	5,2
3	8,1	7,8
4	10,8	10,4
5	13,5	13,0
6	16,2	15,6
7	18,9	18,2
8	21,6	20,8
9	24,3	23,4

	25
1	2,5
2	5,0
3	7,5
4	10,0
5	12,5
6	15,0
7	17,5
8	20,0
9	22,5

	24	23
1	2,4	2,3
2	4,8	4,6
3	7,2	6,9
4	9,6	9,2
5	12,0	11,5
6	14,4	13,8
7	16,8	16,1
8	19,2	18,4
9	21,6	20,7

	22	21
1	2,2	2,1
2	4,4	4,2
3	6,6	6,3
4	8,8	8,4
5	11,0	10,5
6	13,2	12,6
7	15,4	14,7
8	17,6	16,8
9	19,8	18,9

N.	L. 0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	22	21
1	2,2	2,1
2	4,4	4,2
3	6,6	6,3
4	8,8	8,4
5	11,0	10,5
6	13,2	12,6
7	15,4	14,7
8	17,6	16,8
9	19,8	18,9

	20
1	2,0
2	4,0
3	6,0
4	8,0
5	10,0
6	12,0
7	14,0
8	16,0
9	18,0

	.19
1	1,9
2	3,8
3	5,7
4	7,6
5	9,5
6	11,4
7	13,3
8	15,2
9	17,1

	18
1	1,8
2	3,6
3	5,4
4	7,2
5	9,0
6	10,8
7	12,6
8	14,4
9	16,2

	17
1	1,7
2	3,4
3	5,1
4	6,8
5	8,5
6	10,2
7	11,9
8	13,6
9	15,3

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	**18**
252	40 140	157	175	192	209	226	243	261	278	295	1 1,8
253	312	329	346	364	381	398	415	432	449	466	2 3,6
254	483	500	518	535	552	569	586	603	620	637	3 5,4
255	654	671	688	705	722	739	756	773	790	807	4 7,2
256	824	841	858	875	892	909	926	943	960	976	5 9,0
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	6 10,8
258	41 162	179	196	212	229	246	263	280	296	313	7 12,6
259	330	347	363	380	397	414	430	447	464	481	8 14,4
											9 16,2
260	497	514	531	547	564	581	597	614	631	647	
261	664	681	697	714	731	747	764	780	797	814	**17**
262	830	847	863	880	896	913	929	946	963	979	1 1,7
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	2 3,4
264	42 160	177	193	210	226	243	259	275	292	308	3 5,1
											4 6,8
265	325	341	357	374	390	406	423	439	455	472	5 8,5
266	488	504	521	537	553	570	586	602	619	635	6 10,2
267	651	667	684	700	716	732	749	765	781	797	7 11,9
268	813	830	846	862	878	894	911	927	943	959	8 13,6
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	9 15,3
270	43 136	152	169	185	201	217	233	249	265	281	
271	297	313	329	345	361	377	393	409	425	441	**16**
272	457	473	489	505	521	537	553	569	584	600	1 1,6
273	616	632	648	664	680	696	712	727	743	759	2 3,2
274	775	791	807	823	838	854	870	886	902	917	3 4,8
											4 6,4
275	933	949	965	981	996	*012	*028	*044	*059	*075	5 8,0
276	44 091	107	122	138	154	170	185	201	217	232	6 9,6
277	248	264	279	295	311	326	342	358	373	389	7 11,2
278	404	420	436	451	467	483	498	514	529	545	8 12,8
279	560	576	592	607	623	638	654	669	685	700	9 14,4
280	716	731	747	762	778	793	809	824	840	855	
281	871	886	902	917	932	948	963	979	994	*010	**15**
282	45 025	040	056	071	086	102	117	133	148	163	1 1,5
283	179	194	209	225	240	255	271	286	301	317	2 3,0
284	332	347	362	378	393	408	423	439	454	469	3 4,5
											4 6,0
285	484	500	515	530	545	561	576	591	606	621	5 7,5
286	637	652	667	682	697	712	728	743	758	773	6 9,0
287	788	803	818	834	849	864	879	894	909	924	7 10,5
288	939	954	969	984	*000	*015	*030	*045	*060	*075	8 12,0
289	46 090	105	120	135	150	165	180	195	210	225	9 13,5
290	240	255	270	285	300	315	330	345	359	374	
291	389	404	419	434	449	464	479	494	509	523	**14**
292	538	553	568	583	598	613	627	642	657	672	1 1,4
293	687	702	716	731	746	761	776	790	805	820	2 2,8
294	835	850	864	879	894	909	923	938	953	967	3 4,2
											4 5,6
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	5 7,0
296	47 129	144	159	173	188	202	217	232	246	261	6 8,4
297	276	290	305	319	334	349	363	378	392	407	7 9,8
298	422	436	451	465	480	494	509	524	538	553	8 11,2
299	567	582	596	611	625	640	654	669	683	698	9 12,6
300	712	727	741	756	770	784	799	813	828	842	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	15
304	287	302	316	330	344	359	373	387	401	416	1 1,5
305	430	444	458	473	487	501	515	530	544	558	2 3,0
306	572	586	601	615	629	643	657	671	686	700	3 4,5
307	714	728	742	756	770	785	799	813	827	841	4 6,0
308	855	869	883	897	911	926	940	954	968	982	5 7,5
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 9,0
310	49 136	150	164	178	192	206	220	234	248	262	7 10,5
311	276	290	304	318	332	346	360	374	388	402	8 12,0
312	415	429	443	457	471	485	499	513	527	541	9 13,5
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	14
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	1 1,4
317	50 106	120	133	147	161	174	188	202	215	229	2 2,8
318	243	256	270	284	297	311	325	338	352	365	3 4,2
319	379	393	406	420	433	447	461	474	488	501	4 5,6
320	515	529	542	556	569	583	596	610	623	637	5 7,0
321	651	664	678	691	705	718	732	745	759	772	6 8,4
322	786	799	813	826	840	853	866	880	893	907	7 9,8
323	920	934	947	961	974	987	*001	*014	*028	*041	8 11,2
324	51 055	068	081	095	108	121	135	148	162	175	9 12,6
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	
328	587	601	614	627	640	654	667	680	693	706	13
329	720	733	746	759	772	786	799	812	825	838	1 1,3
330	851	865	878	891	904	917	930	943	957	970	2 2,6
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	3 3,9
332	52 114	127	140	153	166	179	192	205	218	231	4 5,2
383	244	257	270	284	297	310	323	336	349	362	5 6,5
334	375	388	401	414	427	440	453	466	479	492	6 7,8
335	504	517	530	543	556	569	582	595	608	621	7 9,1
336	634	647	660	673	686	699	711	724	737	750	8 10,4
337	763	776	789	802	815	827	840	853	866	879	9 11,7
338	892	905	917	930	943	956	969	982	994	*007	
339	53 020	033	046	058	071	084	097	110	122	135	
340	148	161	173	186	199	212	224	237	250	263	12
341	275	288	301	314	326	339	352	364	377	390	1 1,2
342	403	415	428	441	453	466	479	491	504	517	2 2,4
343	529	542	555	567	580	593	605	618	631	643	3 3,6
344	656	668	681	694	706	719	732	744	757	769	4 4,8
345	782	794	807	820	832	845	857	870	882	895	5 6,0
346	908	920	933	945	958	970	983	995	*008	*020	6 7,2
347	54 033	045	058	070	083	095	108	120	133	145	7 8,4
348	158	170	183	195	208	220	233	245	258	270	8 9,6
349	283	295	307	320	332	345	357	370	382	394	9 10,8
350	407	419	432	444	456	469	481	494	506	518	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
350	54 497	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	**13**
355	55 023	035	047	060	072	084	096	108	121	133	1 | 1,3
356	145	157	169	182	194	206	218	230	242	255	2 | 2,6
357	267	279	291	303	315	328	340	352	364	376	3 | 3,9
358	388	400	413	425	437	449	461	473	485	497	4 | 5,2
359	509	522	534	546	558	570	582	594	606	618	5 | 6,5
360	630	642	654	666	678	691	703	715	727	739	6 | 7,8
361	751	763	775	787	799	811	823	835	847	859	7 | 9,1
362	871	883	895	907	919	931	943	955	967	979	8 | 10,4
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9 | 11,7
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	**12**
366	348	360	372	384	396	407	419	431	443	455	1 | 1,2
367	467	478	490	502	514	526	538	549	561	573	2 | 2,4
368	585	597	608	620	632	644	656	667	679	691	3 | 3,6
369	703	714	726	738	750	761	773	785	797	808	4 | 4,8
370	820	832	844	855	867	879	891	902	914	926	5 | 6,0
371	937	949	961	972	984	996	*008	*019	*031	*043	6 | 7,2
372	57 054	066	078	089	101	113	124	136	148	159	7 | 8,4
373	171	183	194	206	217	229	241	252	264	276	8 | 9,6
374	287	299	310	322	334	345	357	368	380	392	9 | 10,8
375	403	415	426	438	449	461	473	484	496	507	
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	**11**
379	864	875	887	898	910	921	933	944	955	967	1 | 1,1
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	2 | 2,2
381	58 092	104	115	127	138	149	161	172	184	195	3 | 3,3
382	206	218	229	240	252	263	274	286	297	309	4 | 4,4
383	320	331	343	354	365	377	388	399	410	422	5 | 5,5
384	433	444	456	467	478	490	501	512	524	535	6 | 6,6
385	546	557	569	580	591	602	614	625	636	647	7 | 7,7
386	659	670	681	692	704	715	726	737	749	760	8 | 8,8
387	771	782	794	805	816	827	838	850	861	872	9 | 9,9
388	883	894	906	917	928	939	950	961	973	984	
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	**10**
391	218	229	240	251	262	273	284	295	306	318	1 | 1,0
392	329	340	351	362	373	384	395	406	417	428	2 | 2,0
393	439	450	461	472	483	494	506	517	528	539	3 | 3,0
394	550	561	572	583	594	605	616	627	638	649	4 | 4,0
395	660	671	682	693	704	715	726	737	748	759	5 | 5,0
396	770	780	791	802	813	824	835	846	857	868	6 | 6,0
397	879	890	901	912	923	934	945	956	966	977	7 | 7,0
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	8 | 8,0
399	60 097	108	119	130	141	152	163	173	184	195	9 | 9,0
400	206	217	228	239	249	260	271	282	293	304	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	**11**
407	959	970	981	991	*002	*013	*023	*034	*045	*055	1 1,1
408	61 066	077	087	098	109	119	130	140	151	162	2 2,2
409	172	183	194	204	215	225	236	247	257	268	3 3,3
410	278	289	300	310	321	331	342	352	363	374	4 4,4
411	384	395	405	416	426	437	448	458	469	479	5 5,5
412	490	500	511	521	532	542	553	563	574	584	6 6,6
413	595	606	616	627	637	648	658	669	679	690	7 7,7
414	700	711	721	731	742	752	763	773	784	794	8 8,8
415	805	815	826	836	847	857	868	878	888	899	9 9,9
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	**10**
422	531	542	552	562	572	583	593	603	613	624	1 1,0
423	634	644	655	665	675	685	696	706	716	726	2 2,0
424	737	747	757	767	778	788	798	808	818	829	3 3,0
425	839	849	859	870	880	890	900	910	921	931	4 4,0
426	941	951	961	972	982	992	*002	*012	*022	*033	5 5,0
427	63 043	053	063	073	083	094	104	114	124	134	6 6,0
428	144	155	165	175	185	195	205	215	225	236	7 7,0
429	246	256	266	276	286	296	306	317	327	337	8 8,0
430	347	357	367	377	387	397	407	417	428	438	9 9,0
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	**9**
438	147	157	167	177	187	197	207	217	227	237	1 0,9
439	246	256	266	276	286	296	306	316	326	335	2 1,8
440	345	355	365	375	385	395	404	414	424	434	3 2,7
441	444	454	464	473	483	493	503	513	523	532	4 3,6
442	542	552	562	572	582	591	601	611	621	631	5 4,5
443	640	650	660	670	680	689	699	709	719	729	6 5,4
444	738	748	758	768	777	787	797	807	816	826	7 6,3
445	836	846	856	865	875	885	895	904	914	924	8 7,2
446	933	943	953	963	972	982	992	*002	*011	*021	9 8,1
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	**10**
458	66 087	096	106	115	124	134	143	153	162	172	1 1,0
459	181	191	200	210	219	229	238	247	257	266	2 2,0
460	276	285	295	304	314	323	332	342	351	361	3 3,0
461	370	380	389	398	408	417	427	436	445	455	4 4,0
462	464	474	483	492	502	511	521	530	539	549	5 5,0
463	558	567	577	586	596	605	614	624	633	642	6 6,0
464	652	661	671	680	689	699	708	717	727	736	7 7,0
465	745	755	764	773	783	792	801	811	820	829	8 8,0
466	839	848	857	867	876	885	894	904	913	922	9 9,0
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	**9**
473	486	495	504	514	523	532	541	550	560	569	1 0,9
474	578	587	596	605	614	624	633	642	651	660	2 1,8
475	669	679	688	697	706	715	724	733	742	752	3 2,7
476	761	770	779	788	797	806	815	825	834	843	4 3,6
477	852	861	870	879	888	897	906	916	925	934	5 4,5
478	943	952	961	970	979	988	997	*006	*015	*024	6 5,4
479	68 034	043	052	061	070	079	088	097	106	115	7 6,3
480	124	133	142	151	160	169	178	187	196	205	8 7,2
481	215	224	233	242	251	260	269	278	287	296	9 8,1
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	**8**
488	842	851	860	869	878	886	895	904	913	922	1 0,8
489	931	940	949	958	966	975	984	993	*002	*011	2 1,6
490	69 020	028	037	046	055	064	073	082	090	099	3 2,4
491	108	117	126	135	144	152	161	170	179	188	4 3,2
492	197	205	214	223	232	241	249	258	267	276	5 4,0
493	285	294	302	311	320	329	338	346	355	364	6 4,8
494	373	381	390	399	408	417	425	434	443	452	7 5,6
495	461	469	478	487	496	504	513	522	531	539	8 6,4
496	548	557	566	574	583	592	601	609	618	627	9 7,2
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	**9**
507	501	509	518	526	535	544	552	561	569	578	1 0,9
508	586	595	603	612	621	629	638	646	655	663	2 1,8
509	672	680	689	697	706	714	723	731	740	749	3 2,7
510	757	766	774	783	791	800	808	817	825	834	4 3,6
511	842	851	859	868	876	885	893	902	910	919	5 4,5
512	927	935	944	952	961	969	978	986	995	*003	6 5,4
513	71 012	020	029	037	046	054	063	071	079	088	7 6,3
514	096	105	113	122	130	139	147	155	164	172	8 7,2
515	181	189	198	206	214	223	231	240	248	257	9 8,1
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	**8**
522	767	775	784	792	800	809	817	825	834	842	1 0,8
523	850	858	867	875	883	892	900	908	917	925	2 1,6
524	933	941	950	958	966	975	983	991	999	*008	3 2,4
525	72 016	024	032	041	049	057	066	074	082	090	4 3,2
526	099	107	115	123	132	140	148	156	165	173	5 4,0
527	181	189	198	206	214	222	230	239	247	255	6 4,8
528	263	272	280	288	296	304	313	321	329	337	7 5,6
529	346	354	362	370	378	387	395	403	411	419	8 6,4
530	428	436	444	452	460	469	477	485	493	501	9 7,2
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	**7**
538	73 078	086	094	102	111	119	127	135	143	151	1 0,7
539	159	167	175	183	191	199	207	215	223	231	2 1,4
540	239	247	255	263	272	280	288	296	304	312	3 2,1
541	320	328	336	344	352	360	368	376	384	392	4 2,8
542	400	408	416	424	432	440	448	456	464	472	5 3,5
543	480	488	496	504	512	520	528	536	544	552	6 4,2
544	560	568	576	584	592	600	608	616	624	632	7 4,9
545	640	648	656	664	672	679	687	695	703	711	8 5,6
546	719	727	735	743	751	759	767	775	783	791	9 6,3
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	1 0,8
563	75 051	059	066	074	082	089	097	105	113	120	2 1,6
564	128	136	143	151	159	166	174	182	189	197	3 2,4
565	205	213	220	228	236	243	251	259	266	274	4 3,2
566	282	289	297	305	312	320	328	335	343	351	5 4,0
567	358	366	374	381	389	397	404	412	420	427	6 4,8
568	435	442	450	458	465	473	481	488	496	504	7 5,6
569	511	519	526	534	542	549	557	565	572	580	8 6,4
											9 7,2
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	7
582	492	500	507	515	522	530	537	545	552	559	1 0,7
583	567	574	582	589	597	604	612	619	626	634	2 1,4
584	641	649	656	664	671	678	686	693	701	708	3 2,1
585	716	723	730	738	745	753	760	768	775	782	4 2,8
586	790	797	805	812	819	827	834	842	849	856	5 3,5
587	864	871	879	886	893	901	908	916	923	930	6 4,2
588	938	945	953	960	967	975	982	989	997	*004	7 4,9
589	77 012	019	026	034	041	048	056	063	070	078	8 5,6
											9 6,3
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	**8**
608	390	398	405	412	419	426	433	440	447	455	1 0,8
609	462	469	476	483	490	497	504	512	519	526	2 1,6
610	533	540	547	554	561	569	576	583	590	597	3 2,4
611	604	611	618	625	633	640	647	654	661	668	4 3,2
612	675	682	689	696	704	711	718	725	732	739	5 4,0
613	746	753	760	767	774	781	789	796	803	810	6 4,8
614	817	824	831	838	845	852	859	866	873	880	7 5,6
615	888	895	902	909	916	923	930	937	944	951	8 6,4
616	958	965	972	979	986	993	*000	*007	*014	*021	9 7,2
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	**7**
623	449	456	463	470	477	484	491	498	505	511	1 0,7
624	518	525	532	539	546	553	560	567	574	581	2 1,4
625	588	595	602	609	616	623	630	637	644	650	3 2,1
626	657	664	671	678	685	692	699	706	713	720	4 2,8
627	727	734	741	748	754	761	768	775	782	789	5 3,5
628	796	803	810	817	824	831	837	844	851	858	6 4,2
629	865	872	879	886	893	900	906	913	920	927	7 4,9
630	934	941	948	955	962	969	975	982	989	996	8 5,6
631	80 003	010	017	024	030	037	044	051	058	065	9 6,3
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	**6**
638	482	489	496	502	509	516	523	530	536	543	1 0,6
639	550	557	564	570	577	584	591	598	604	611	2 1,2
640	618	625	632	638	645	652	659	665	672	679	3 1,8
641	686	693	699	706	713	720	726	733	740	747	4 2,4
642	754	760	767	774	781	787	794	801	808	814	5 3,0
643	821	828	835	841	848	855	862	868	875	882	6 3,6
644	889	895	902	909	916	922	929	936	943	949	7 4,2
645	956	963	969	976	983	990	996	*003	*010	*017	8 4,8
646	81 023	030	037	043	050	057	064	070	077	084	9 5,4
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	7
662	086	092	099	105	112	119	125	132	138	145	1 0,7
663	151	158	164	171	178	184	191	197	204	210	2 1,4
664	217	223	230	236	243	249	256	263	269	276	3 2,1
665	282	289	295	302	308	315	321	328	334	341	4 2,8
666	347	354	360	367	373	380	387	393	400	406	5 3,5
667	413	419	426	432	439	445	452	458	465	471	6 4,2
668	478	484	491	497	504	510	517	523	530	536	7 4,9
669	543	549	556	562	569	575	582	588	595	601	8 5,6
670	607	614	620	627	633	640	646	653	659	666	9 6,3
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	6
683	442	448	455	461	467	474	480	487	493	499	1 0,6
684	506	512	518	525	531	537	544	550	556	563	2 1,2
685	569	575	582	588	594	601	607	613	620	626	3 1,8
686	632	639	645	651	658	664	670	677	683	689	4 2,4
687	696	702	708	715	721	727	734	740	746	753	5 3,0
688	759	765	771	778	784	790	797	803	809	816	6 3,6
689	822	828	835	841	847	853	860	866	872	879	7 4,2
690	885	891	897	904	910	916	923	929	935	942	8 4,8
691	948	954	960	967	973	979	985	992	998	*004	9 5,4
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	
705	819	825	831	837	844	850	856	862	868	874	
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	**7**
708	85 003	009	016	022	028	034	040	046	052	058	1 | 0,7
709	065	071	077	083	089	095	101	107	114	120	2 | 1,4
											3 | 2,1
710	126	132	138	144	150	156	163	169	175	181	4 | 2,8
711	187	193	199	205	211	217	224	230	236	242	5 | 3,5
712	248	254	260	266	272	278	285	291	297	303	6 | 4,2
713	309	315	321	327	333	339	345	352	358	364	7 | 4,9
714	370	376	382	388	394	400	406	412	418	425	8 | 5,6
											9 | 6,3
715	431	437	443	449	455	461	467	473	479	485	
716	491	497	503	509	516	522	528	534	540	546	
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	**6**
723	914	920	926	932	938	944	950	956	962	968	1 | 0,6
724	974	980	986	992	998	*004	*010	*016	*022	*028	2 | 1,2
											3 | 1,8
725	86 034	040	046	052	058	064	070	076	082	088	4 | 2,4
726	094	100	106	112	118	124	130	136	141	147	5 | 3,0
727	153	159	165	171	177	183	189	195	201	207	6 | 3,6
728	213	219	225	231	237	243	249	255	261	267	7 | 4,2
729	273	279	285	291	297	303	308	314	320	326	8 | 4,8
											9 | 5,4
730	332	338	344	350	356	362	368	374	380	386	
731	392	398	404	410	415	421	427	433	439	445	
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	
737	747	753	759	764	770	776	782	788	794	800	**5**
738	806	812	817	823	829	835	841	847	853	859	1 | 0,5
739	864	870	876	882	888	894	900	906	911	917	2 | 1,0
											3 | 1,5
740	923	929	935	941	947	953	958	964	970	976	4 | 2,0
741	982	988	994	999	*005	*011	*017	*023	*029	*035	5 | 2,5
742	87 040	046	052	058	064	070	075	081	087	093	6 | 3,0
743	099	105	111	116	122	128	134	140	146	151	7 | 3,5
744	157	163	169	175	181	186	192	198	204	210	8 | 4,0
											9 | 4,5
745	216	221	227	233	239	245	251	256	262	268	
746	274	280	286	291	297	303	309	315	320	326	
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
749	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	
765	366	372	377	383	389	395	400	406	412	417	
766	423	429	434	440	446	451	457	463	468	474	
767	480	485	491	497	502	508	513	519	525	530	
768	536	542	547	553	559	564	570	576	581	587	
769	593	598	604	610	615	621	627	632	638	643	
770	649	655	660	666	672	677	683	689	694	700	
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	
783	376	382	387	393	398	404	409	415	421	426	
784	432	437	443	448	454	459	465	470	476	481	
785	487	492	498	504	509	515	520	526	531	537	
786	542	548	553	559	564	570	575	581	586	592	
787	597	603	609	614	620	625	631	636	642	647	
788	653	658	664	669	675	680	686	691	697	702	
789	708	713	719	724	730	735	741	746	752	757	
790	763	768	774	779	785	790	796	801	807	812	
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
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P. P.

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906	713	718	722	727	732	737	742	746	751	756	
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908	809	813	818	823	828	832	837	842	847	852	
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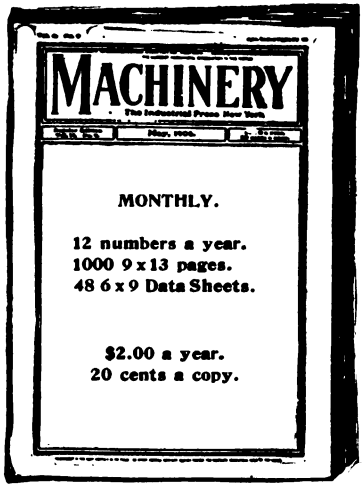

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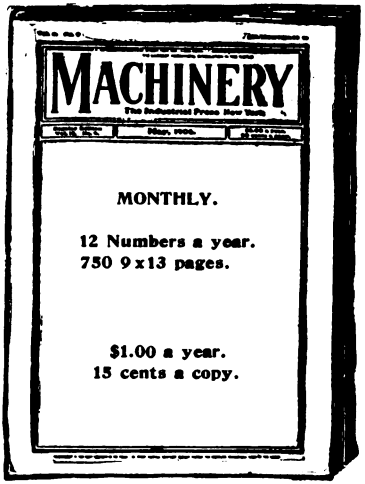

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